PERTURBATIONS AHEAD OF A WING MOVING IN A STRATIFIED FLUID

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An interesting nonlinear effect in liquid wave motion is that there can be considerable perturbations far ahead of a steadily moving source under certain conditions. It has been found [1] that a nonstationary wave sequence is formed ahead of a ship model if the model's speed is in a certain range around the critical velocity $c_* = \sqrt{gH}$ (H is depth of the liquid g is acceleration due to gravity). With a stratified liquid, the surface mode is accompanied by a set of internal ones, for each of which one can define a critical velocity that sets an upper bound to the propagation speed for infinitely small harmonic perturbations on that mode. In the stationary-wave class, it can be exceeded for example by solitary waves or bores, for which it is the lower bound.

Further, experiments have been made with a wing moving near the critical velocity for one of the internal modes. There was vertical velocity shear in the unperturbed state. There are about 20 papers on this effect, of which the most informative are [2-5]. The perturbations ahead of a source can also be related to other factors. For example, in [6], they were due to a strong pulse arising in a liquid when a cylinder accelerates from rest, and in [7], to the complicated form of motion for a cylinder.

The experiments were performed in the channel shown schematically in Fig. 1, which was 5 m long, 0.2 m wide, and 0.6 m high; it was filled at the bottom with a mixture of glycerol and water having density ρ_1 and above this with distilled water having density $\rho_3 < \rho_1$. Then the solution and water were set in motion in opposite direction with velocities u_1 and u_3 averaged over the depths. A stable interlayer was formed with characteristic thickness $h_2 << h_1$, h_3 and density $\rho_3 < \rho_2 < \rho_1$. To provide control of h_2, there were the adjustable plates 1, 3, and 9 (Fig. 1).

The lower layer was set in motion by the rotating cylinder 8, which was placed unsymmetrically in the gap between the base 6 and the false base 7. The perforated plate 5 and guides 4 equalized the velocity. The upper layer was driven by a blade pump via equalizing devices lying outside the working part of the channel. We examined only states of motion stable under uncontrolled perturbations. Controlled perturbations were introduced by the symmetrical wing 2 having diameter for the middle section D = 2 cm and aspect ratio 6:1, which was mounted on three holders with diameter 3 mm and moved with velocity U at a distance h = const from the free surface. The characteristic time of acceleration to a state of steady motion was about 0.1 sec.

The speed of the liquid u was determined from the time deformation in the mark formed by a floating crystal of malachite green. Line 1 in Fig. 2 shows the typical u(y). Capillary effects at small u_3 caused the liquid to adhere not only to the bottom but also to the free surface indicated by the triangle in Figs. 1, 3, and 4.

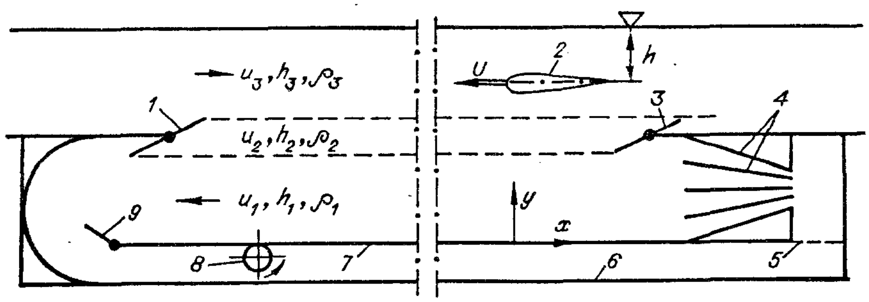

Fig. 1

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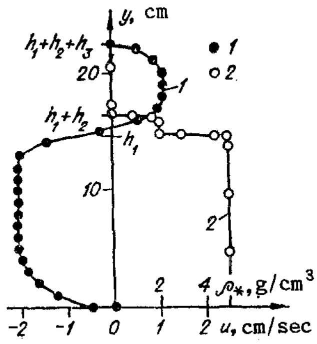

Fig. 2

The $\rho(y)$ distribution was derived by means of a laser on the basis that the beam propagates in a medium whose refractive index n varies only along y, for which $n_1 \sin\theta_1 = n_2 \sin\theta_2$ for any points 1 and 2 (θ is the angle between the tangent to the ray path and the y axis). If n_1 and θ_1 are known, one determines n_2 at point 2 by measuring θ_2, which can be done in various ways. We converted $n(y)$ to $\rho(y)$ on the basis of a linear relation between them for small variations in ρ. Figure 2 shows a typical $\rho(y)$ (line 2, $\rho_* = (\rho-\rho_3)\cdot 10^3$), where there are two prominent steps, whose distance apart gave h_2. There are no such steps in the velocity profile. This interesting effect occurs because the molecular viscosity exceeds the trace-component diffusion coefficient by several orders of magnitude when ρ varies.

The perturbations introduced by the wing were visualized by photography of a luminous screen bearing a set of inclined lines through the liquid layer. In the sharp-gradient zones (steps), there was characteristic distortion in the inclined lines (Figs. 3 and 4).

In [4], there is a detailed theoretical analysis of the critical velocities for the various modes. Under our conditions, a good approximation is provided by a three-layer liquid, with the free surface replaced by a solid lid. Then the critical velocities are defined by the roots of

$$\rho_1\rho_3 b_1^2 b_3^2 h_2 + \rho_1\rho_2 b_1^2 b_2^2 h_3 + \rho_2\rho_3 b_2^2 b_3^2 h_1 - \rho_1 b_1^2 h_2 h_3 g\Delta_2 - \\ - \rho_3 b_3^2 h_1 h_2 g\Delta_1 - \rho_2 b_2^2 h_1 h_3 g(\Delta_1 + \Delta_2) + h_1 h_2 h_3 g^2 \Delta_1\Delta_2 = 0,$$

in which $b_i = u_i - c$ (c is the critical velocity and i = 1, 2, 3); $\Delta_1 = \rho_1-\rho_2$; $\Delta_2 = \rho_2 - \rho_3$. The formula gives four roots $c_1^{\pm}$ and $c_2^{\pm}$, with the subscript referring to the number of the internal mode and the superscript to the perturbation propagation direction.

The Fig. 3 photographs were recorded with h_1, h_2, h_3 = 14.4; 1.5; 6.8 cm, ρ_1, ρ_2, ρ_3 = 1.005; 1.002; 1.000 g/cm³, u_1, u_3 = −2.0; 1.0 cm/sec, h = 9.3 cm. For $u_2 = 0$, the calculated values are c_1^+, c_1^-, c_2^+, c_2^- = 4.58; −4.74; 1.25; −1.15 cm/sec. For picture a $U/c_2^+ = 1$, and the time from the start of motion is t = 100 ± 2 sec. Ahead of the wing, there are considerable nonstationary perturbations, which run ahead by about 15D. They take the form of solitary waves as a hump on the upper boundary and a depression on the lower boundary of the intermediate layer.

Appreciable perturbations ahead of the wing were observed also near the critical velocities for the various modes. The widths of the corresponding intervals were dependent on the parameters. In this example, the perturbations ahead of the wing were appreciable for $0.75 < U/c_2^+ < 1.6$. Pictures b and c show how the perturbations behave at the upper bound to this range. In b, $U/c_2^+ = 1.45$, and although there is a marked perturbation ahead of the wing, it is only slightly ahead of it. In c, $U/c_2^+ = 1.73$, and there are no perturbations ahead of the wing. Behind the wing, at this speed and higher ones (up to a certain region around the critical velocity c_1), one gets monoharmonic waves with synchronous oscillation in the upper and lower bounds to the intermediate layer.

Perturbations ahead of the wing were observed when it moved with near-critical velocities in the intermediate layer or in the upper layer and when the direction of motion changed. They occurred also when the liquid was at rest in the unperturbed state, as Fig. 4 shows, for

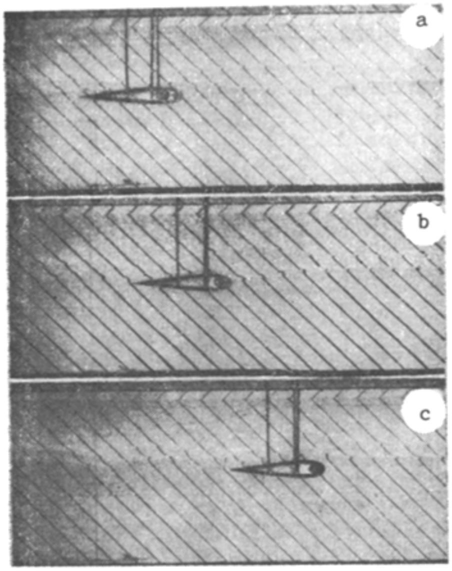

Fig. 3

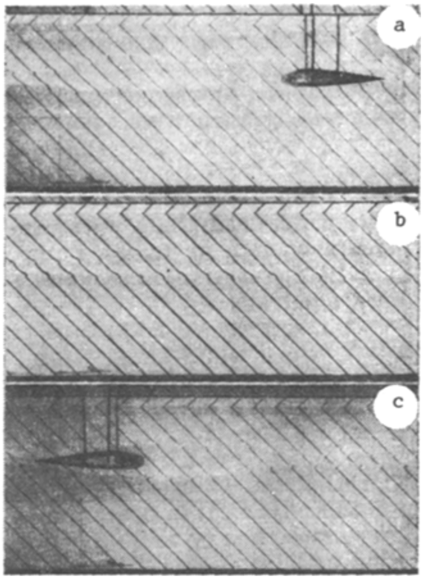

Fig. 4

which $u_1 = u_2 = u_3 = 0$, with the densities as in Fig. 3, and h_1, h_2, h_3 = 14.5; 1.3; 6.9 cm, $c_1^{\pm}$, $c_2^{\pm}$ = ±4.84; ±1.19 cm/sec. In a, $U = c_2^- = -1.19$ cm/sec, with the wing moving to the left in the intermediate layer, and t = 125 ± 2 sec. There is no symmetry between the perturbations ahead of the wing at the upper and lower bounds in the layer, which is due to differing density steps at those boundaries and to slight assymmetry in relation to them in the wing path. Behind the wing, the layer becomes much thinner, which illustrates how these effects are related to blocking.

In b, $U/c_2^- = 0.82$, t = 120 ± 5 sec, and the leading edge of the perturbation runs ahead of the wing by more than 25D and three humps are formed on the upper boundary. In c, $U/c_2^+ = 1.2$, t = 87 ± 2 sec, and the speed with which the leading edge runs ahead of the wing is substantially reduced. There is also a change in the asymmetry in the perturbations at the upper and lower boundaries.

The [3] results were qualitatively confirmed by the perturbations with the wing moving with speeds in the region of c_1 in the presence or absence of velocity shear between the layers.

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